1996-03

Modeling light valve replacements in a training unit : the spare demand process

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THESIS

MODELING LIGHT VALVE REPLACEMENTS IN A TRAINING UNIT: THE SPARE DEMAND PROCESS.

by

Michael A. Hollister

March, 1996

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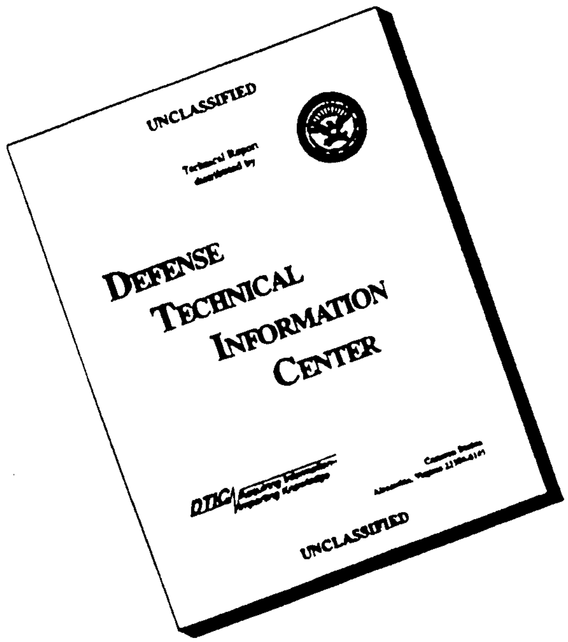

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REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*
2. REPORT DATE March 1996
3. REPORT TYPE AND DATES COVERED Master's Thesis
4. TITLE AND SUBTITLE MODELING LIGHT VALVE REPLACEMENTS IN A TRAINING UNIT: THE SPARE DEMAND PROCESS.
5. FUNDING NUMBERS
6. AUTHOR(S) HOLLISTER, MICHAEL A.
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey CA 93943-5000
8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
10. SPONSORING/MONITORING AGENCY REPORT NUMBER
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(maximum 200 words)*

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14. SUBJECT TERMS Allocation Models, Spares Usage Statistics, One-Hundred per cent sparing
15. NUMBER OF PAGES 55
16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18 298-102

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MODELING LIGHT VALVE REPLACEMENTS IN A TRAINING UNIT: THE SPARE DEMAND PROCESS.

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Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
March, 1996

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ABSTRACT

This thesis utilizes data on the time to failure of individual light valves to represent the random process of all failures at a many-valve training unit. This process governs the demand for replacement spares. The light valve replacement model, with finite spares, is based on theoretical results concerning the Poisson tendency for a superposition of renewal processes. Graphical analysis and a simulation verify that the theory can apply under practical circumstances. The model is distinguished by its applicability for use in standard spreadsheets; no specialized statistical features are required.

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EXECUTIVE SUMMARY

A. PURPOSE

The purpose of this thesis is to develop a model requested by the Department of the Navy's Naval Air Systems Command (PMA-205), which determines the estimated time for its current inventory of spare light valves to be exhausted. The estimated time for all spares to fail is based on light valves' historical failure data and on renewal theory arguments. Once developed, two allocation systems for the inventory of spares are presented.

B. BACKGROUND

Martin Marietta Corporation stopped production of the Talaria Light Valve used in U.S. Navy and Marine Corps training simulators as a video display device. The effect of this loss of production precipitated the question of the life expectancy of the Navy's stockpiles of ready for installation (RFI) light valves (approximately 184 as of October 6, 1995).

A repair facility has been contracted to refurbish non-ready for installation (Non-RFI) or burnt light valves to replace the current stockpiles of RFI light valves as they are depleted. The repair facility, Vacuum Optics, located in Tucson, Arizona is under contract to repair light valves at a cost of $8,050.00 per tube. In order to plan for purchasing refurbished light valves, a model is needed to estimate the rate of failures of Talaria Light Valves.

C. RESULTS

A quantitative rationale for estimating spare valve consumption time has been based on the following steps. Historical data on times to failure of each type of light valve may be used to estimate each type's time-to-failure distribution. A non-parametric representation has been employed since evidence of a bimodal distribution for some valve types rules out a simple parametric (Weibull) model. Then future successive failures of valves in the sockets of a training device at a particular site are estimated to occur according to the above distribution and are assumed to form a renewal process for each socket. The superposition of these renewal processes represents the demand for spares by all sockets of a unit at a site, and is shown to be approximately a Poisson process. Use of this information allows the distribution of time to consume a fixed number of spares to be estimated; it is approximately of gamma form. That distribution can then guide spare provisioning at sites. For example, a policy of 100% sparing (maintaining at least one spare for each socket at a site) can control the risk that a site will fail to meet its operational schedule.

A complication arises in that repaired or refurbished valves appear to have a shorter time to failure than do members of the original set. This information is, to date, based on experience with only six repaired valves. If repaired bulbs are blended with original bulbs, the spares consumption rate will increase and the spares allocation policy must be reexamined. That reexamination is beyond the scope of this thesis.

I. INTRODUCTION

A. BACKGROUND

On December 15, 1993, Martin Marietta Corporation announced its intentions to stop production of the General Electric Projector and the Talaria Light Valve used in U.S. Navy and Marine Corps training simulators as video display devices. The effect of this loss of production precipitated the question of the life expectancy of the Navy's stockpiles of ready for installation (RFI) light valves (approximately 184 as of October 1995). The Talaria Light Valve is advertised by its manufacturer to last approximately 4,000 hours. However, historical data from simulator sites has shown this value to be variable and somewhat overstated in actual operation.

The inventory of light valves is under the direct supervision of the Aviation Supply Office (ASO) Philadelphia, Pa., which has contracted with a repair facility to refurbish non-ready for installation (Non-RFI), or burnt, light valves to replace the current stockpiles of RFI light valves as they are depleted. The repair facility, Vacuum Optics, located in Tucson Arizona currently repairs light valves at a contracted cost of $8,050.00 per tube. To plan for purchasing refurbished light valves, a method is needed to estimate the rate of failures of Talaria Light Valves. This thesis develops a statistical approach (based on a Poisson Process model) to predict the life of spare Talaria light valves available at a training location.

B. OBJECTIVES

This thesis has three primary objective areas. First, it *models the distribution of lifetimes of Talaria Light Valves*. Parametric and non-parametric options are considered. Next, *it suggests two alternative <u>spares allocation models</u> that are easily adaptable to a spreadsheet* such as Lotus or Excel. The allocation models presented make it easier to predict and track spare lifetimes and to provide an estimate of the date on which the inventory of spare Talaria Light Valves will be exhausted. Finally, *it provides suggestions for extending the life of light valves.*

The modeling of lifetimes is based on theoretical results concerning the superposition of renewal processes, and upon a simulation verification that the theory should apply under practical circumstances, i.e., when a training unit has finitely many valves in simultaneous use.

C. SCOPE

This thesis will deal directly with the following two questions.

1. Can we estimate, with some level of accuracy, how long the current inventory of spare light valves will last?
2. Do simpler system models exist for spares usage?

Current historical data, as provided by the simulator operators, is in the form of operating times since installation for burning light valves, and failure times for burnt light valves waiting for refurbishment; no distinction between the sockets in which light valves were installed is made. Using these historical data, this thesis provides simple formulae that can be applied at the operational level as a predictor of spares utilization. User requested requirements of this analysis are, *any methodology developed must be conformable to a spreadsheet, use historical quarterly data as provided by the sites, and be easy to apply at the operational level.*

The model that has been developed avoids the complexities of fitting a theoretical distribution of the data which would make computations and understanding difficult. Throughout this thesis a set of failure-time data (hours of operation from installation until failure) obtained from the Miramar training unit in Appendix A will be utilized for examples.

D. PREVIEW

Chapter II. Two alternative model options are considered for lifetime distributions. The parametric approach is based on a bimodal Weibull distribution while the non-parametric method is based on using given data to empirically fit a distribution.

Chapter III. A stochastic model based on renewal theory (superposition of renewal processes) is presented. This model is the basis for deriving estimations of the spares failure rate. The rate is calculated from the approximately exponential inter-failure times (times between consecutive failures in a multiple socket system).

Chapter IV. System performance measures are derived; these are used to estimate the time at which all spares burn out, and the approximate number of spares required to reach a target year.

Chapter V. Two alternative system allocation models are proposed. The reasons why they are improvements over the current system are given.

Chapter VI. Results, Conclusions, and Recommendations for extending the lives of light valves are provided.

Appendixes. A listing of available failure-time data for various light-valve types and training unit locations. Simulation results and graphs validate the renewal theory results about superposition of various numbers of simulator sockets.

E. LIGHT VALVE SYSTEM DESCRIPTION

The following light valve operating system description and notation are essential in understanding the presented analysis:

- **Sites (S):** There are currently nine operational sites (Miramar, El Toro, Beaufort, Oceana, Whidbey, Lemoore, Yuma, Cherry Point and Cecil Field) that use Talaria Light Valves.

- **Light Valve types (L):** Light Valves are of four different varieties (G32p, G38, G39 and G43). They are operated independently of one another and are identical within types.

- **Number of Sockets (H):** H is the number of light valve sockets of type L at site S.

The single-gun light valve is a sealed vacuum tube containing a deformable oil-based fluid. It uses an external arc lamp as a light source. Optically, it is similar to a slide or movie projector. Light valves of different types are not interchangeable. The G43 and G32p variety are used in single light valve (SLV) projectors. The G38 and G39 light valves operate, as pairs, in multiple light valve projectors.

A light valve is replaced for two reasons. The first is a determination, made prior to actual mechanical failure, by technical experts that the projector video output in the simulator has degraded to unacceptable levels. Cavitating bubbles on the video display caused by the viscous breakdown of the oil-based film in the light valve is the primary indicator of this impending failure. The second reason is infrequent mechanical failures which are due to either poor light valve workmanship or improper handling. Light valve failure updates are sent to ASO on a quarterly basis. The data contain the recorded time of replacement (in hours) for non-RFI light valves that failed at the site, and the usage time of light valves that are still operational at the time the report is made.

II. MODELING LIGHT VALVE TIMES TO FAILURE

A. PARAMETRIC FAILURE TIME MODEL

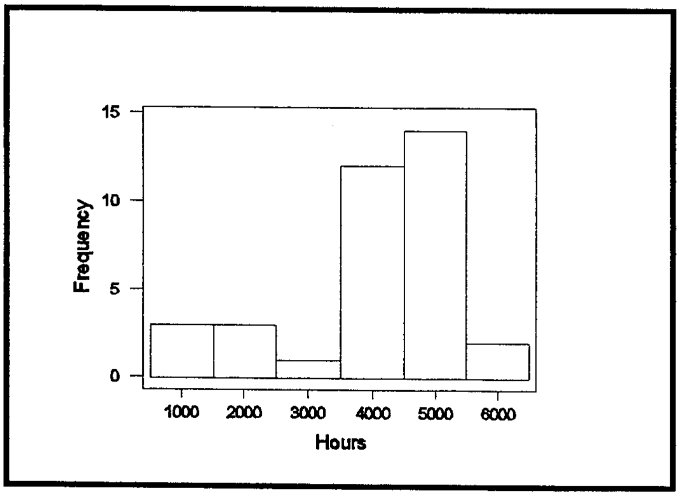

Figure 1. The histogram of the thirty-five failure times from Miramar (Appendix A), has two modes or humps. Notice that the breakpoint of the two modes is at approximately 3500 hours. This is an initial indication that a single mode parametric form may not accurately fit the underlying distribution of the data.

A histogram is widely used as an initial exploratory data analysis tool. In Figure 1, the histogram reveals a bimodal (two modes) tendency of the thirty-five Miramar failure times. An explanation for this bimodal tendency is Martin-Marietta's announcement of its impending shutdown of Talaria light valve production led to the immediate loss of key personnel and a drop in morale. These factors caused subsequent light valves produced to be of lower quality than the original light valves which averaged four thousand hours.The Weibull distribution is often appropriately used to model failure times of electronic components. However, the Weibull density has only one mode; a fit of the Weibull to the G32P is only crudely satisfactory.

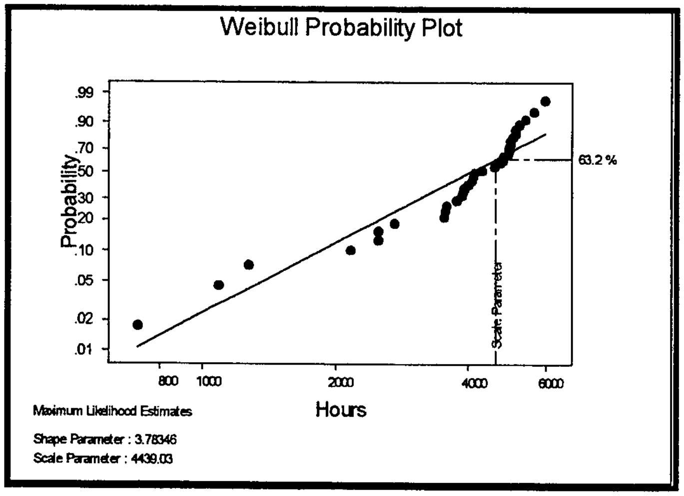

Figure 2. This graph is a plot of the thirty-five Miramar failure times on Weibull paper. Data from a Weibull distribution will plot along the straight line. In our case, only the data points less than 3500 hours are along this line. Values larger than 3500 hours appear to lie along a different line.

Further analysis for this data set is performed with a Weibull Probability Plot (see Figure 2) to show the initial guess of a bimodal distribution is justified. The suspected mixing of light valve grades at Miramar cannot be accounted for using a single mode parametric form.

An approach to model bimodal data as recommended by Law and Kelton (1991), is to split the data into two parts based on the histogram (for the above data this would be at 3500 hours), and try a Weibull fit to each half. This splitting technique will lead to a model of the following form:

$$p \times W(\alpha_1, \beta_1) + (1\text{-}p) \times W(\alpha_2, \beta_2) \qquad \textbf{(1)}$$

This model has five parameters to be estimated from the data (p, α_1, β_1, α_2, β_2). Where p (the breakpoint) is the fraction of failure times smaller than 3500 hours. The alphas (α_1 and α_2) and betas (β_1 and β_2) are the two sets of Weibull shape and scale parameters when fitted to each half of the data. This parametric approach has the following drawbacks which make it an inconvenient option at the operational level:

1. A plot of the data provides ***no strong physical motivation*** for any particular parametric form.

2. The five parameters in Equation (1) must be estimated for each light valve type using a curve fitting procedure called maximum likelihood estimators. This procedure ***requires computational effort*** that is time consuming for a personal computer. Computations (means, variances, etc.) for a five-parameter model would be ***difficult to implement in a spreadsheet.***

3. The ***estimate of p may be inaccurate.*** Once the failure times are split into parts, there is no clear way of knowing if data from the shorter life light valves may actually belong to the longer life population, and vice versa. It is unclear how this inaccuracy affects the final assessment (of time to system consumption of spares).

B. NON-PARAMETRIC MODEL

A non-parametric or distribution-free model (see Figure 3 below) can give quite efficient approximations in circumstances such that no suitable parametric distribution is known or usable. Also importantly, these approximations can be understood and implemented by nontechnically oriented users. The simplest non-parametric model for the distribution of individual light valve failure times is obtained by constructing the empirical cumulative distribution function (ECDF) from existing failure data. This process is easy to explain and can be carried out on small computers. The empirical cumulative distribution function can be smoothed if desired, but for a reasonable amount of data (n = 20 or more failure times) this is unnecessary.

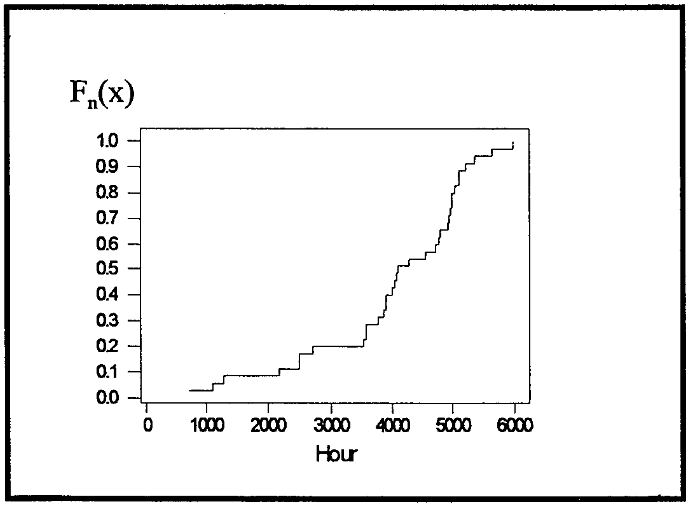

Figure 3. The Empirical Cumulative Distribution Function (ECDF) provides insights into the general shape and skewness of data fitted using non-parametric analysis. The steepness of the slope from 3500 hours to 5000 hours indicates that Miramar experienced a disproportionate number of failures in this range.

We can define a ECDF from data representations $X_1, X_2, \ldots, X_n$ as

$$F_n(x) = \frac{No.\ of\ X_i s \leq x}{n} \tag{2}$$

This formula avoids the many problems in Equation (1). The ECDF is used as an exploratory tool in this analysis to:

1. Graphically compare an estimate of the true distribution function of our data with the distribution function of one of the fitted distributions in probability.
2. Randomly generate failure times in a simulation.
3. Gain insight into the general shape and skewness of the underlying distribution.

In this thesis, representations of the form of Equation (2) are used to generate the ECDF reciprocal classically known as the empirical survivor function (ESF). The ESF will be used to estimate the mean remaining life of a light valve which has lasted to some time (t). Equation (2) is also used to describe the individual valve failure times, and then to describe the H-socket *system* failure times. The use of ECDF to describe failure-time variability is conservative in that it makes minimal assumptions about the failure process. It does assume that future failure times statistically resemble those from the past, and that they are independent samples from a time-invariant process, i.e., that there is no trend in the failure process. It also requires that a minimal amount of historical data be available.

III. DEMAND FOR SPARE BULBS: A STOCHASTIC MODEL

A. TIMES TO SYSTEM FAILURE

Provided $\mathbf{T}_i$ is a random variable representing the time between successive failures and **R** is a random variable representing the lifetime of a light valve, the decision-aid information in this thesis can be supported by the following facts of probability theory.

Theorem 1.

If, in a renewal process, the inter-arrival times, $\mathbf{T}_i$s, are independent and exponentially distributed with mean $1 / \lambda$, then the renewal process is a Poisson process with rate λ (Ross, 1993).

Theorem 2.

The superposition of a large number H, of independent renewal processes having the same failure time distribution is asymptotically a Poisson Process with rate

$$\lambda = \frac{H}{E[\boldsymbol{R}]}. \tag{3}$$

See Karlin and Taylor (1975, pp.221-223).

An estimate of λ is

$$\hat{\lambda} = \frac{H}{\hat{E}[\boldsymbol{R}]} \tag{4}$$

Here,

$$\hat{E}[\mathbf{R}] = \sum_{i=1}^{n} \frac{r_i}{n} \tag{5}$$

is an estimate of the expected mean failure time between renewals, i.e., individual failure times, where the r_i s are the realized values of **R**, namely the observed failure times. Any system studied consists of a training unit equipped with H light valve sockets (see Figure 4 below). The light valves are kept under conditions that allow them to fail apparently independently after varying operating times. Each socket gives rise to a sequence of renewal (replacement) times (realizations of **R**) where: $R_i(1)$, $R_i(1) + R_i(2)$, $R_i(1) + R_i(2) + R_i(3)$, . . .represent the times at which light valves occupying socket/hole i (i=1, 2, . . . , H) are renewed or replaced.

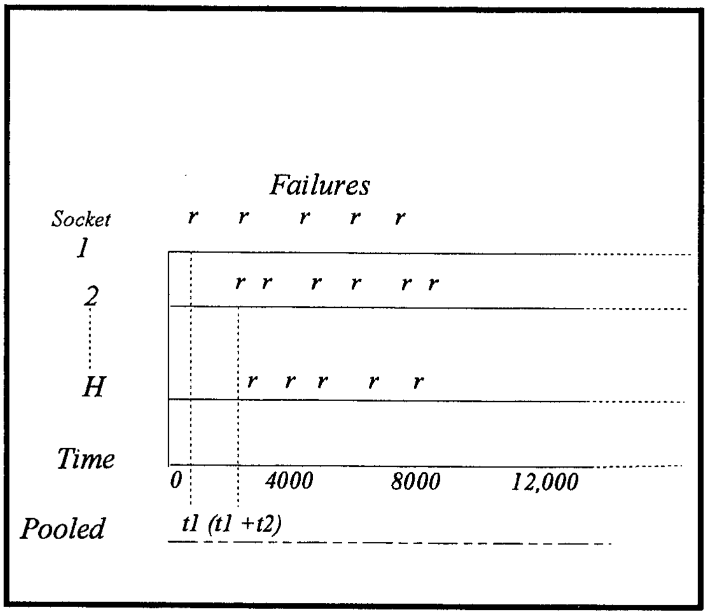

Figure 4. With r representing an actual failure time, the depicted pooled process shows that the time the first light valve fails $r_1(1)$ is the shortest time in the process. We call this initial failure t(1). The next smallest failure $r_2(1)$ in the system occurs as the first failure from socket 2. If we add t(1), the time the first valve failed, to t(2), the time between the first and second failures, we have t(1) + t(2) as the second smallest system failure time. We can continue in this manner adding inter-failure times for the entire process.

The system renewal/replacement times are the merged (pooled) times of replacement for the individual sockets/holes. For instance, if all H sockets/holes start with new light valves then T(1), the time to first system failure is the minimum of (the first) H failure times, i.e.,

$$\mathbf{T(1)} = \min(\, \mathbf{R_1(1)},\ \mathbf{R_2(1)}, \ldots, \mathbf{R_H(1)}). \qquad \mathbf{(6)}$$

If $R_2(1)$ turns out to be the minimum failure time, then it is replaced by a light valve with life time $R_2(2)$, and the time to second system failure is

$$\mathbf{T(1) + T(2)} = \min(\mathbf{R_1(1)}, \ \mathbf{R_2(1) + R_2(2)}, \ \mathbf{R_3(1)}, \dots, \mathbf{R_H(1)}) \tag{7}$$

and so on.

B. SUPERPOSITION OF RENEWAL PROCESSES SIMULATION: RESULTS

The applicability of Theorem 2, for finite H, can be tested for accuracy with a simulation (see Appendix B) to see if *the pooling together of data from a finite number of sockets at a site resembles a Poisson Process.* Three checks can be made from the simulated inter-failure times ($\mathbf{T_i}$**s**) to validate the model.

1. Graph the system inter-failure times. Now examine if the histogram of the simulated times is approximately exponential.

2. Compute the mean and standard deviation of system inter-failure times. The <u>standard deviation and mean of the inter-failure times should be approximately the same</u>.

3. Confirm that for the system, <u>the simulation failure rate is approximately H / $E[\hat{\mathbf{R}}]$</u>.

The data in Appendix A, of pooled G32p failures at Miramar, were used in the simulation. The accuracy of the model was checked with H=2, 5, and 8 pooled sockets. Results of simulation system inter-failure times are summarized below.

No. Pooled Sockets	Estimated Mean Number of Failures per 10,000 hrs. Theory Results / Simulation Results			Simulation Statistics for System Inter-Failures Mean	Standard Deviation
2	5.11	/	4.96	2016.5	1269.5
5	12.79	/	12.06	828.7	716
8	20.46	/	19.28	518.6	491.5

Table 1. The estimated mean number of failures in 10,000 hours is computed through theoretical and simulation computations. The table highlights that the simulation failure rate which is the reciprocal of the mean number of failures is approximately the same as provided by the theoretical computations regardless of the number of sockets pooled. Notice that the values for the mean and standard deviation gradually get closer as the number of sockets increases. This correlates with a Poisson distribution.

1. In Appendixes C, D, and E note that the histogram appears to *indicate that the underlying distributions are better approximated by the exponential distribution as H increases*. The straightness of the quantile-quantile exponential plots also confirms the increased validity of an exponential assumption for increasing H.

2. The mean and standard deviation of inter-failure times in Table 1 is an additional affirmation of result (1).

3. The theoretical and simulation failure rates (also in Table 1) are approximately the same in the three cases.

We conclude that the superposition of system renewal times to create a sequence of consecutive system inter-failures may be adequately represented by a Poisson process. If anything the Poisson projection is even more variable than is suggested by the simulation, since the simulation indicates that the standard deviation of system inter-failure times is somewhat smaller than their mean.

Example 1. **Computation of the estimated failure rate $\hat{\lambda}$.**

Using the Miramar G32p data set in Appendix A, where the r_i s are the 35 historical failure times in column (1) and H = 10 is the number of installed sockets in column (2), we can compute the estimated failure rate as follows:

1. Compute the estimate of the expected mean failure time:

$$\hat{E}[\mathbf{R}] = \sum_{i=1}^{n} \frac{r_i}{n} = 3908.7,$$

2. Now compute the estimate of the failure rate as

$$\hat{\lambda} = \frac{H}{\hat{E}[\boldsymbol{R}]} = 10/3908.7 = 0.002558.$$

This says light valves will fail at a rate of 0.002558 light valves per hour or 25.58 light valves per 10,000 hours, with random variability described by the Poisson distribution; for example the variance is

$$\hat{\mathrm{Var}}[\mathbf{R}] = \hat{E}[\mathbf{R}]. \tag{8}$$

IV. ESTIMATION OF SYSTEM PERFORMANCE MEASURES

A. SPARES USAGE STATISTICS

Two important estimations of spares usage can now be found using historical data and previously discussed formulas. They are:

1. The estimate of the mean time to spares depletion.

2. The estimate of the expected number of spares needed until some specified time t.

The estimate of the mean time to use n spares and the corresponding variance are two statistics needed to answer various probability questions about the approximate time at which the spares will be exhausted. If $\mathbf{X}_i$ is a random variable representing the life of spare i and n is the total number of spares, the two statistics can be estimated as follows:

$\mathbf{X}_1$ = the time to failure of the first spare,

$\mathbf{X}_2$ = the time to failure of the second spare,

.

.

.

$\mathbf{X}_n$ = the time to failure of the n^{th} spare.

Then

$$\mathbf{S}_n = \mathbf{X}_1 + \mathbf{X}_2 + \ldots + \mathbf{X}_n \tag{9}$$

represents the waiting time until n spares have failed. Since each spare will be used in the pooled process (which we have shown to be approximately a Poisson Process), they are assumed to be installed and to fail at the same rate as the non-RFI valves. We call that rate λ. Then using results in Theorems 1 and 2, the random variable $\mathbf{S}_n$ has the gamma distribution, since a sum of n independent exponentials is distributed as Gamma (n , λ). The gamma distribution has mean = n / λ and variance = n / λ^2. The formulae to compute estimates of the mean and variance are as follows:

$$\hat{E}[\mathbf{S_n}] = (n / \hat{\lambda}) = n \times (\hat{E}[\mathbf{R}] / H), \tag{10}$$

and

$$\hat{Var}[\mathbf{S_n}] = (n / \hat{\lambda}^2) = n \times (\hat{E}[\mathbf{R}] / H)^2. \tag{11}$$

The estimate of the expected number of spares needed is based on the number of failures in some time t. We can make use of one of the most important results of Renewal Theory to provide a formula to estimate the number of failures in some arbitrary time t. From Ross (1993), $\mathbf{N}(t)$ (the number of renewals in some time t) is Poisson distributed. The expected number of renewals in some time t is $E[\mathbf{N}(t)] = \lambda t$. The estimate of this expected value is

$$\hat{E}[\mathbf{N}(t)] = \hat{\lambda} t = (H / \hat{E}[\mathbf{R}]) \times t. \tag{12}$$

Since the mean and variance are the same for a Poisson distribution, we have

$$\hat{Var}[N(t)] = \hat{\lambda} t = (H / \hat{E}[\mathbf{R}]) \times t. \tag{13}$$

B. PROBABILITY APPROXIMATIONS

We apply Equations (10), (11), (12), and (13) in two important approximations. Since we can approximate a Poisson process with a normal distribution for large t, and the number of renewals, $\mathbf{N}(t)$ has mean $E[\mathbf{N}(t)]$ and variance $Var[\mathbf{N}(t)]$, the first approximation is

$$P(N(t) < n) \approx \Phi\left(\frac{n - E[\hat{N}(t)] - .5}{\sqrt{Var[\hat{N}(t)]}} \right). \tag{14}$$

The number 0.5 in Equation (14) is a continuity correction.

The second approximation can be obtained by noting the following quotation from Ross (1993, pg.305), "*the number of renewals by time t is greater than or equal to n if and only if the nth renewal occurs before or at time t.*" This says that $\mathbf{N}(t) \geq n \Leftrightarrow \mathbf{S}_n \leq t$ and thus the normality assumption for $\mathbf{N}(t)$ applies for S_n with large n. Using estimates from Equations (12) and (13) we now have

$$P(S_n > t) \approx 1 - \Phi\left(\frac{t - \hat{E}[S_n]}{\sqrt{\hat{Var}[S_n]}}\right). \tag{15}$$

Based on approximations (14) and (15), we can answer many important questions about spares usage. One type of question that can now be answered is in example2.

Example 2. **What is the estimated number of G32p spares needed for Miramar to have 10,000 hours of system operation with 95% reliability?**

Using the Miramar data set in Appendix A and previous computations. We are given:

1. $\hat{E}[\mathbf{R}] = 3908.7$ hrs,
2. H = 10 sockets,
3. t = 10,000 hours .

Now compute the estimates of the mean and variance of $\mathbf{N}(t)$:

$$\hat{E}[\mathbf{N}(t)] = (H / \hat{E}[\mathbf{R}]) \times t = (10/3908.7) \times 10{,}000 = 25.58$$

and the

$$\hat{Var}[\mathbf{N}(t) = \hat{E}[\mathbf{N}(t)] = 25.58.$$

We want

$$.95 \leq P(\mathbf{N}(t) < n)$$

which provides at least 95% reliability of the number of failures in 10,000 hours of system time. We have stated that $\mathbf{N}(t)$ is approximately normal for large t. The above equation can be restated as

$$.95 \le P\left(\frac{N(t) - E(N(t)}{\sqrt{Var(N(t)}} < \frac{n - 25.58 - .5}{\sqrt{25.58}} \right)$$

or in terms of the standard normal random variable $\mathbf{Z}_t$ as

$$.95 \le P\left(\boldsymbol{Z}_t < \frac{n - 25.58 - .5}{\sqrt{25.58}} \right).$$

This in turn gives

$$1.645 = \Phi^{-1}(.95) \le \frac{n - 26.08}{5.06}.$$

Therefore,

$$n \ge (1.645)(5.06) + 26.08 = 34.4$$

So, there is a 95% or higher probability that the number of failures in 10,000 hours, will be less than or equal to 34. Confidence limits can be put on the estimate, but this step is omitted.

C. EXCESS LIFE OF INSTALLED LIGHT VALVES

The following non-parametric calculation is needed to account for the light valves that are still operating. Since their mean remaining life impacts the date at which all spares will burn out.

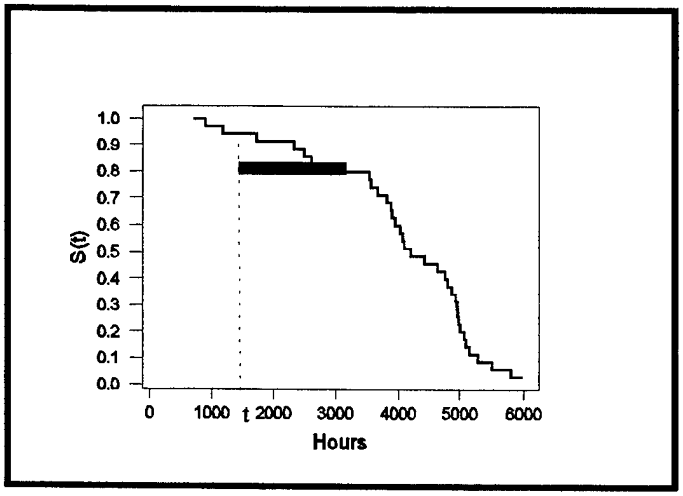

Figure 5. The Empirical Survival Function (ESF) is 1-ECDF. The ESF is used to estimate the mean remaining life of a light valve. We partition the area under the ESF to the right of t (the age of the light valve of interest) into rectangles of height 1/n similar to the one in the figure. Now we sum all the rectangles to the right of t divided by the proportion of data greater than t to find the mean remaining life.

The mean excess life of installed light valves called the conditional mean remaining life of a unit of age t by Barlow and Proschan (1975) can be calculated by using the following formula:

$$E[\mathbf{X} - t|\mathbf{X} > t] = \int_t^{\infty} \frac{(1-\hat{F}(x))dx}{1-\hat{F}(t)} . \qquad \textbf{(16)}$$

One method to approximate this integral is to use a Riemann sum approach of adding together the collection of rectangles which represent the area under the ESF to the right of t (see Figure 5 above). In the empirical survivor function, the failure time that is equal to t or the next largest failure time is assigned the notation x(j). Since this is an empirical survivor function, the height is always 1/n. The base is the difference between the next largest failure time and t. This must be done for all n-(j-1) failure times which are greater than or equal to t (Figure 6 below).

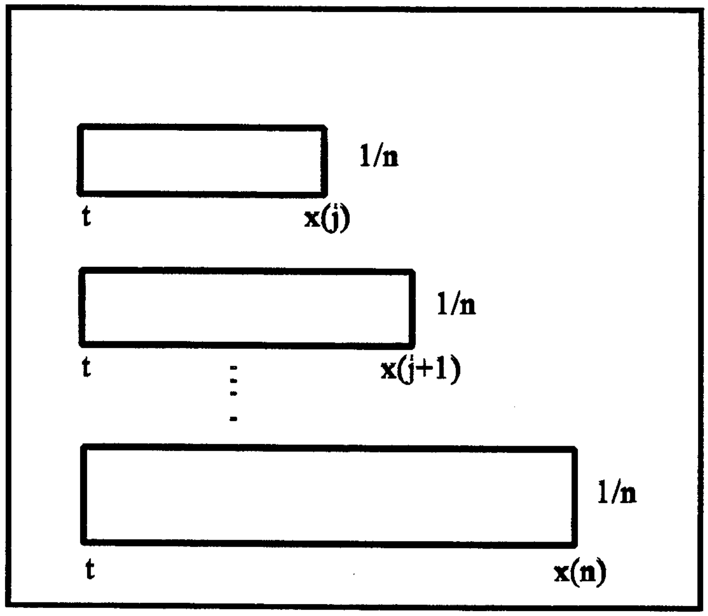

Figure 6. A closer look at Figure 5 reveals that a Riemann sum approach can be applied to add each area.

We can sum the area of each rectangle as follows:

$$(1/n)\times(x_j - t) + (1/n)\times(x_{j+1} - t) + \ldots + (1/n)\times(x_n - t) \qquad \textbf{(17)}$$

Now dividing by the denominator $1-\hat{F}(t)$, in Equation (16), which may be estimated as the proportion of failure times greater than t, we have

$$\frac{(1/n)[(x_j - t) + (x_{j+1} - t) + \ldots + (x_n - t)]}{[n-(j-1)]/n}. \qquad \textbf{(18)}$$

After canceling n in the numerator and denominator, the final form is

$$\frac{(x_j - t) + (x_{j+1} - t) + \ldots + (x_n - t)}{n-(j-1)}. \qquad \textbf{(19)}$$

V. ALLOCATION MODELS

There isn't a current system to track light valve spares consumption rates, or a concrete plan on how many refurbished light valves will be needed. At present, each site maintains its own spares while ASO contracts refurbished light valves based on individual site requisition forms. The problems with this spare's replacement policy include:

1. There is no centralized storage site for spares or refurbished light valves.
2. Individual sites may have unfairly hoarded their own collection of light valves.
3. It has previously been impossible to numerically estimate when all spares will be exhausted.
4. Some sites will start using the shorter-lived refurbished light valves while less needy sites will still have the longer lived spares.

In this chapter, we address two allocation models. Since no site wishes to run out of spares, a policy of maintaining 100% sparing (each site will have at least one spare for each operating socket on hand) will be assumed. The system thus now fails when 100% sparing is compromised. As the current spares are exhausted, this on-site inventory will eventually be exhausted; it may be replaced by refurbished valves.

A. MODEL 1

This system consists of one central (storage and distribution) depot per coast (see Figure 7 on next page), n spares per coast at time zero, H pooled sockets, four light valve types and one repair facility. Each system of several light valves is parallel. A valve in spare status cannot fail. When an operating valve fails it is immediately replaced by a spare. Repairs are assumed to begin immediately. The time until all spares fail is approximately Gamma distributed since the system inter-failure times are approximately exponential with mean equal to that of the original failed light valves divided by H. The introduction of repaired light valves into the system will change that mean.

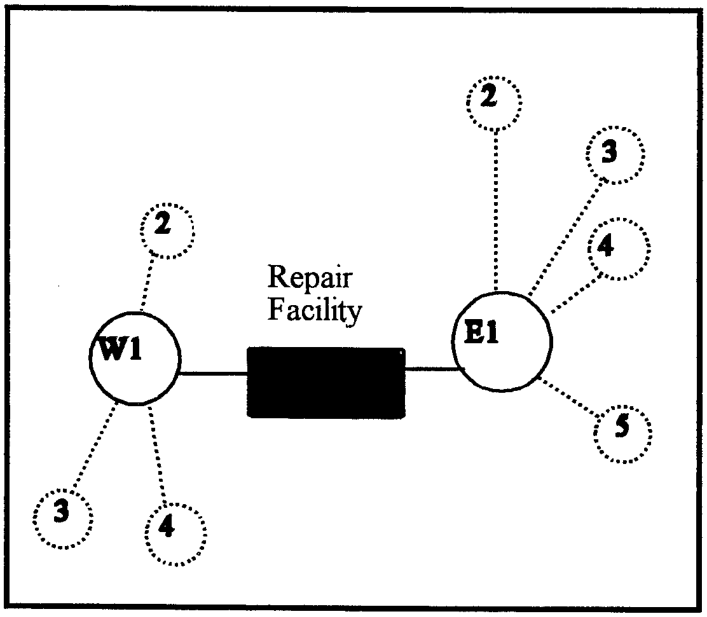

Figure. 7 The network in Model 1 consists of one central hub on each coast. This facility will keep track of spares allocations for the other sites on its coast.

This model offers several attractive features over the old system.

1. Only two sites (I recommend the two largest sites per coast) will contact ASO with requisitions, quarterly updates, etc. Since the entire coast will work as one system all other sites will work directly with the centralized site.

2. Shipments to and from the repair facility should be significantly reduced since they will only come from two sites. The shipments can be scheduled to occur at regular intervals, i.e. monthly and in bulk since we now have a method of predicting failure patterns and a policy of 100% sparing.

3. Shipments between sites can be handled in some cases by military transport (military air, trucks, etc.) instead of using commercial shipments form the repair facility directly to each site.

4. System can be tracked and maintained on a simple spreadsheet.

B. MODEL 2

The model represented in Figure 8 is to have a centralized site near the repair facility handle the system operations.

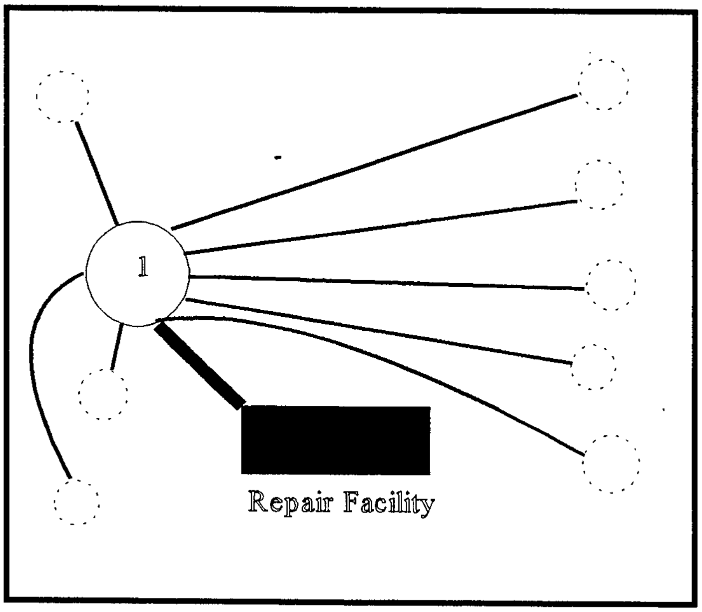

Figure 8. The network in Model 2 has one hub to allocate spares.

The same advantages as in Model 1 are maintained, but shipments from West to East coast may be time consuming even with military transport.

SYSTEM RESULTS FOR MODEL				
Light Valve Type(L)	No. of Spares(n)	No. of sockets(H)	Expected Hours until Spares Depletion	Standard Deviation
G32p	69	48	5,716	688
G43	69	29	8,553	1027.3
G38	26	8	12,772.5	2504.9
G39	20	8	8,767.5	1960.5

Table 2. The estimated time in hours till depletion of spares for each light valve type is computed using the complete data set in Appendix E.

Table 2 provides an estimate of the system time until all spares are depleted. This time can be adjusted to correspond with actual usage hours at the operational level. The table assumes a network like Model 2, where there is one centralized site that stores all spares. Model 1 will have two similar output tables. The data used in the table are from all sites as reported in the quarterly update as of October 6, 1995 (Appendix F).

VI. RESULTS, RECOMMENDATIONS, AND CONCLUSIONS

A. RESULTS

A spreadsheet can be easily developed from these formulae to track light valve spares allocations based on the formulas developed in this thesis. The failure rate must be based on actual historical data. The formula's predictive ability will be significantly enhanced when at least 25 failures of each type of light valve will have occurred. There is some evidence that repaired light valves have a shorter time to failure than the original set, to date the repair facility has only repaired six light valves. The two allocation models offer key savings in shipment cost and centralization of storage and system maintenance. Having one or two sites control all stored valves allows for a more accurate inventory of spares and refurbished valves. The policy of 100% sparing offers a method, based on system failure rates, of planning ahead without taking the risk that a site will fail to meet its operational schedule.

B. RECOMMENDATIONS

1. Alternate Arc Lamps so that older Light Valves use newer Arc Lamps.

<u>Explanation:</u>

An arc lamp generally lasts 1000 hours (at 1000 hours the manufacturer recommends removal to prevent a possible arc lamp explosion). During the first 0-500 hours of life the arc lamp maintains 70% of its luminance. If an arc lamp within this range is paired to a light valve near the end of its theoretical operational life (3000 hours or greater), it could provide adequate performance to offset the effects of a faltering light valve. Alternately, the opposite strategy of coupling a newer light valve (0-2000 hours) with an arc lamp within the last 500 hours of its life is equally valid. Although this has not been tested, conversations with technical representatives at Miramar Air Station confirm the feasibility of this idea. The maintenance time to interchange the arc lamps is approximately four hours.

2. A uniform policy of either operating light valves continuously 4-5 days per week or turning the light valves on and off every day must be established.

Explanation:

It is generally believed that continuous shutdowns and start-ups wear more heavily on electronic components than does continuous running. The light valve is subjected to power spikes with each start-up. Irregular maintenance start-ups can double or triple the weekly wear on the valves. Currently most sites use the daily start-up shutdown method.

3. Install an additional cooling fan in MLV projectors.

Explanation:

Two types of bulbs, the G38 and G39, are used as pairs in a MLV projector. The additional light valve leads to increased temperature and shorter times to failure (probably because degradation of the oil-based film has increased). Many G38 and G39 valves failed around 3000 hours; this is far short of the 4000 hours achieved by G43 and G32p light valves when used in SLV projectors. A cooling fan would bring internal temperatures closer to the range in SLV projectors.

C. CONCLUSIONS

The only long-term fix to the Talaria light valve problem is to find a suitable replacement. The three companies that show the most promise are Ampro, Barco, and Texitron. Barco actually has a product ready to test and at an advantageous cost to the Navy. It is recommended that only enough money be allocated to repairing non-RFI light valves to guarantee 100% sparing since the newer light valve replacement will have a significant cost advantage over paying $8050 for a product whose mean life is only 2500 hous (as advertised by Vacuum Optics).

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APPENDIX A. MIRAMAR G32P DATA SET

COLUMN 1. HISTORICAL FAILURE TIMES (Operational time in hours at failure)		COLUMN 2: INSTALL TIMES (Operational time in hours as of October 6,1995)
4721	5977	45
4085	1271	661
1085	4947	805
4977	2160	939
3861	2714	672
5218	2494	4015
4056	3771	4310
4288	4000	3884
4961	5641	3949
3558	4790	1941
3574	5046	
4801	5372	
710		
3908		
3522		
4573		
4111		
4927		
2497		
5111		
3899		
5101		
4987		

APPENDIX B. SUPERPOSITION SIMULATION: PSEUDOCODE

This pseudocode can be run with simulation software or a statistical package to generate inter-failure times from historical data.

Steps:

1. Construct a piecewise-linear empirical cumulative distribution function (ECDF) from the original data using the smoothed version of Equation (2)

$$F_n(x)=\begin{cases} 0 & \text{if } x<X_{(i)} \\ \dfrac{i-1}{n-1}+\dfrac{x-X_{(i)}}{(n-1)(X_{(i+1)}-X_{(i)})} & \text{if } X_{(i)}\leq x<X_{(i+1)} \\ 1 & \text{if } X_{(n)}\leq x \end{cases}$$

   for i=1, 2, ... , n-1.

   Here the $X_{(i)}$s are the Ordered Statistics. They correspond to the numerical position of the data when sorted from smallest to largest ($X_{(1)}$ is the shortest lifetime and $X_{(n)}$ is the longest).

2. Utilizing an Algorithm by Law and Kelton (1991 pp.495), the following Pseudo Code generates 500 random failure times from the ECDF function to simulate actual failure times for each of the H sockets:

   for f= 1:500 do begin

   for H=1:10 do begin

   Generate U~U(0,1), let P=(n-1)U, and let I= |P| +1

   Return $X_{fH} = X_{(I)} + (P-I+1)(X_{(I+1)}) - X_{(I)})$.

   end

   end

3. Within each of the H sockets, run a cumulative total to add the lifetimes from left to right. Y_{fH}

corresponds to the simulated clock time from time 0 that each $\mathbf{X_{fH}}$ failed.

$$\mathbf{Y_{1H} = X_{1H}}$$

$$\mathbf{Y_{2H} = X_{2H} + X_{1H}}$$

$$\mathbf{Y_{3H} = X_{3H} + X_{2H} + X_{1H}}$$

$$\vdots$$

$$\mathbf{Y_{500H} = X_{500H} + X_{499H} + \ldots + X_{1H}}$$

4. Store all $\mathbf{Y_{fH}}$ in a single 1x(500xH) vector (this pools all sockets into one) and sort from smallest to largest.

$$\mathbf{T_1 = \text{smallest } Y_{fH}}$$

$$\mathbf{T_2 = \text{next largest}}$$

$$\vdots$$

$$\mathbf{T_{500xH} = \text{largest}}$$

5. Compute (500xH) inter-failure times X_i. These times represent the times between the first failure $\mathbf{T_1}$ (regardless of which socket it was in since they are now considered to be one pool) and the second failure $\mathbf{T_2}$ etc. for all light valves.

$$\mathbf{X_1 = T_1}$$

$$\mathbf{X_2 = T_2 - T_1}$$

$$\mathbf{X_3 = T_3 - T_2}$$

$$\vdots$$

$$\mathbf{X_{500xH} = T_{500xH} - T_{(500xH)-1}}$$

APPENDIX C. SUPERPOSITION OF EIGHT G32P SOCKETS

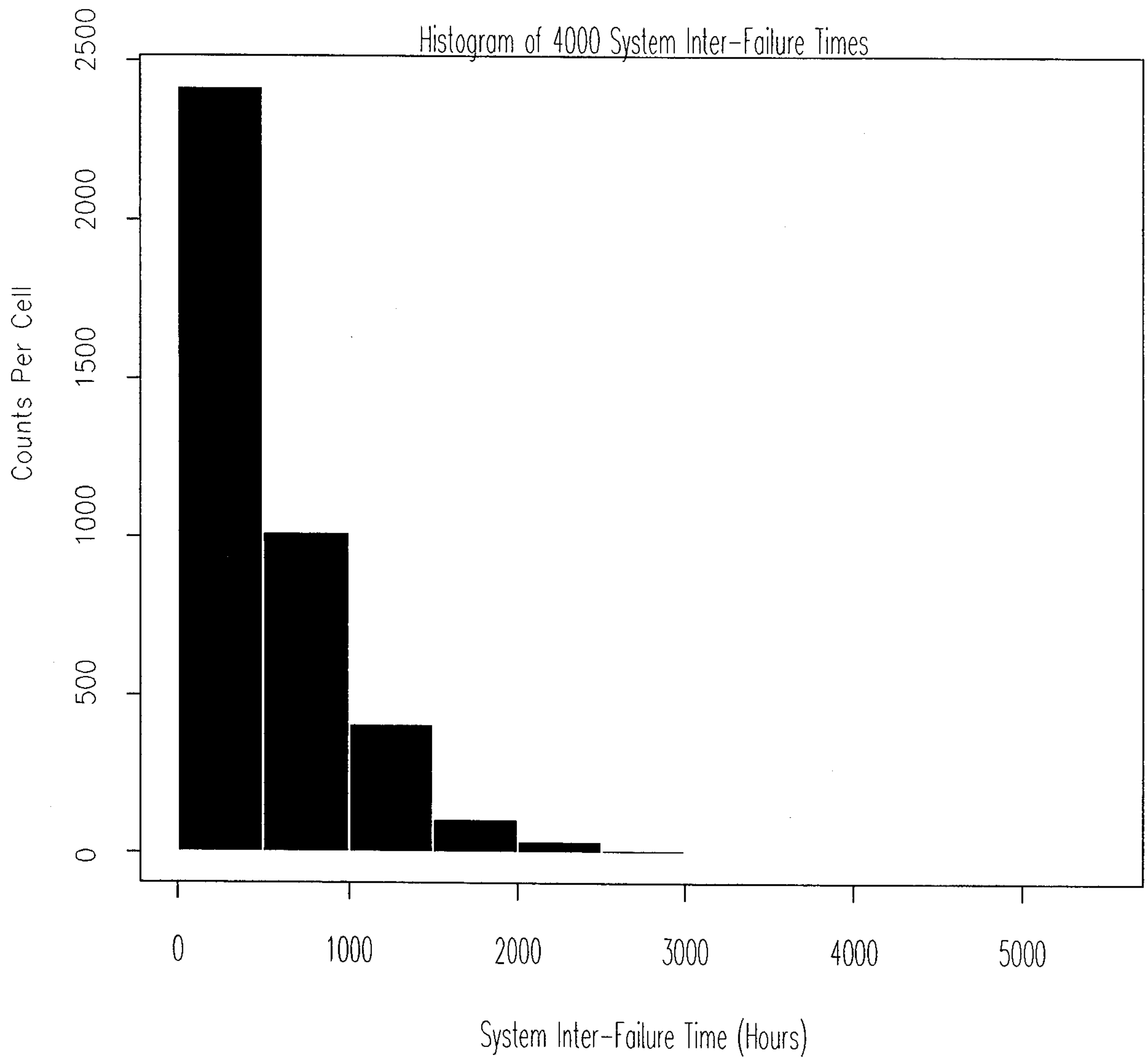

Figure 1. The histogram is a plot of 4000 pooled inter-failure times from a system with eight sockets (500 failures per socket). Its exponential shape is an indication that the pooling of eight sockets at a site, or system of sites, is asymptotically a Poisson Process in accordance with Theorem 2.The histogram is generated from the simulation in Appendix B.

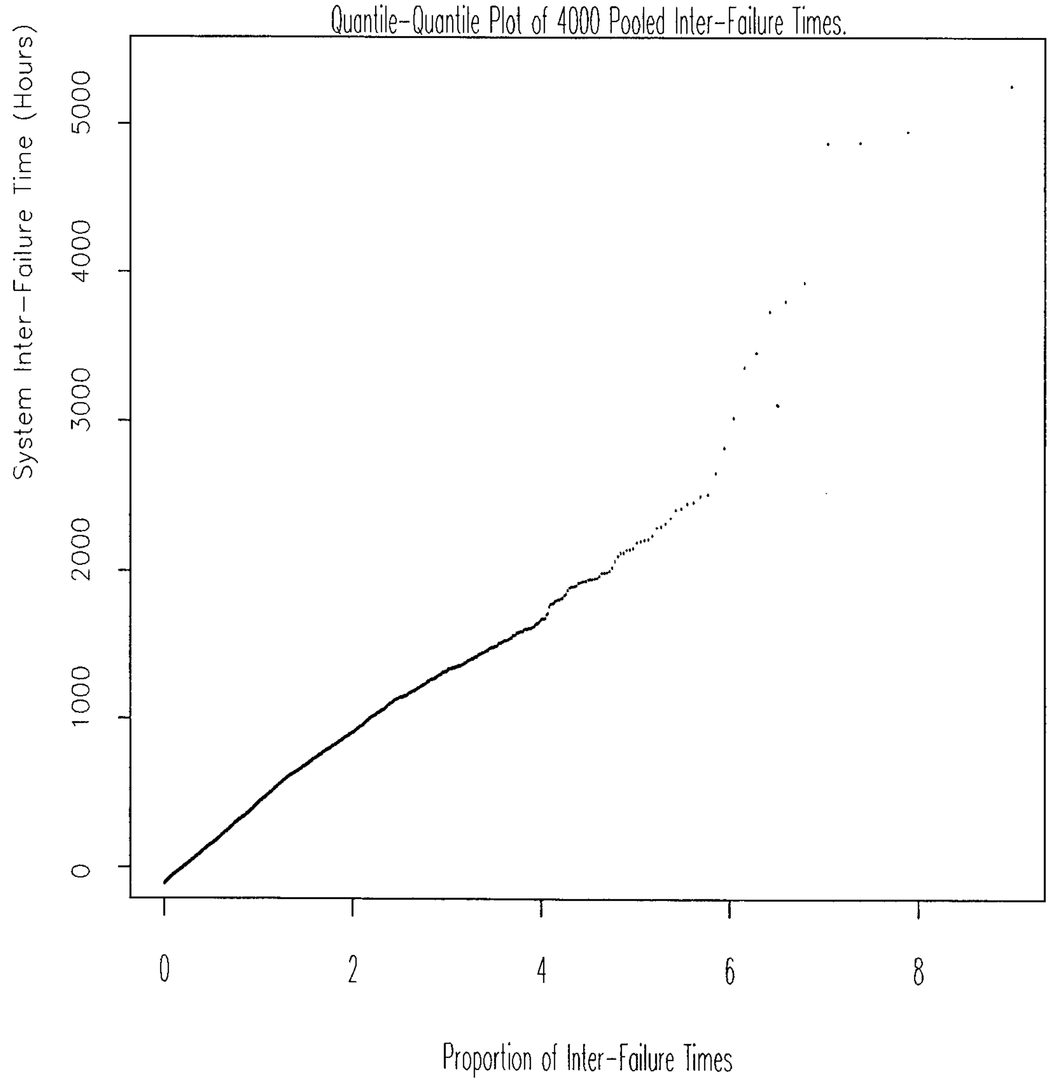

Figure 2.The Quantile-quantile (Q-Q) plot is used to compare the quantiles of the 5000 pooled inter-failure times against the quantiles exponential distribution. If the underlying distribution is exponential the is a straight line. The plot is a near straight line which gives strong confirmation to the assumptions in Figure 1.

APPENDIX D. SUPERPOSITION OF FIVE G32P SOCKETS

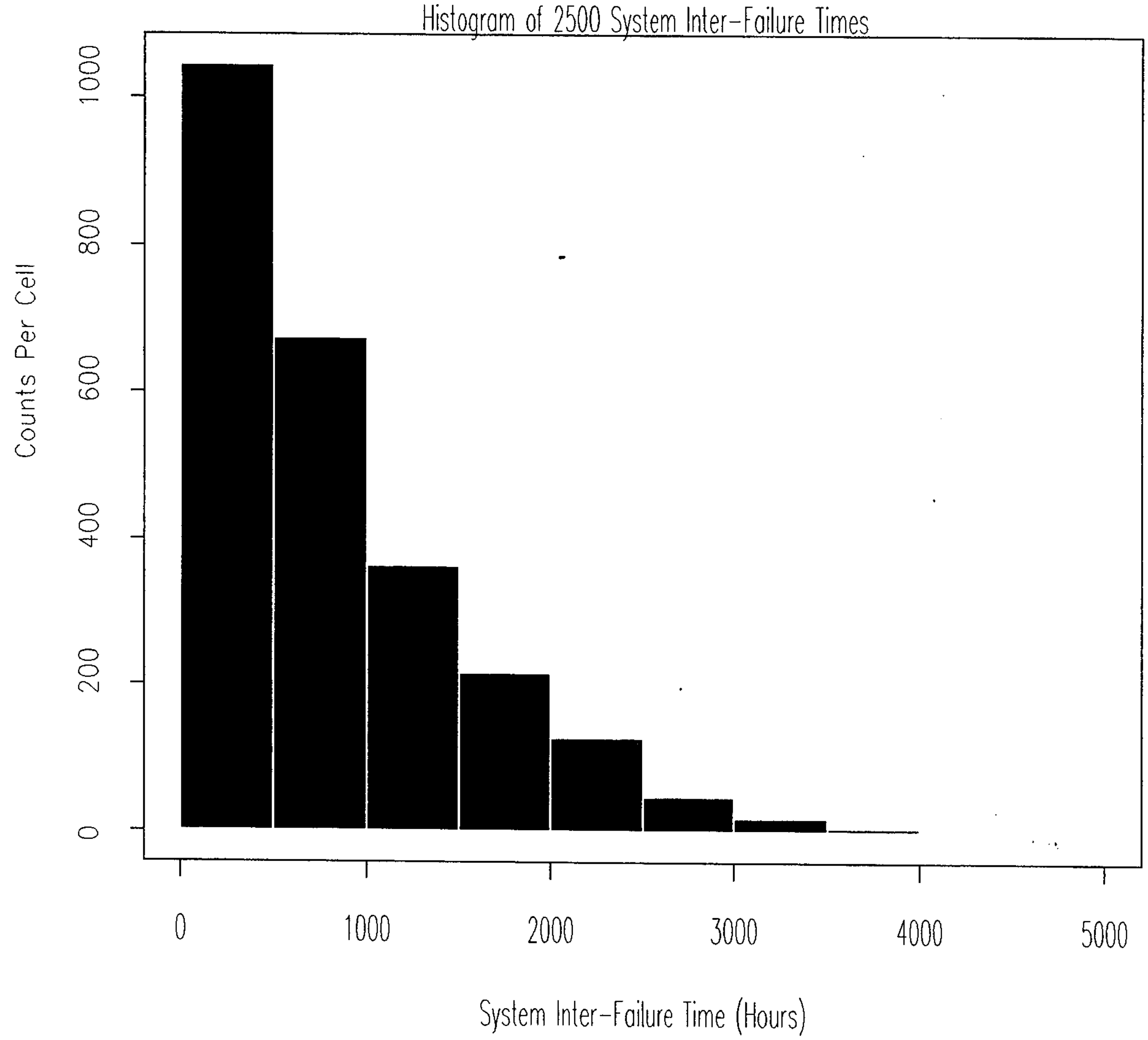

Figure 3. The histogram is a plot of 2500 pooled inter-failure times from a system with five sockets (500 failures per socket). Its strong exponential shape is an indication that the pooling of five sockets at a site, or system of sites, is asymptotically a Poisson Process in accordance with Theorem 2 of Chapter 2.

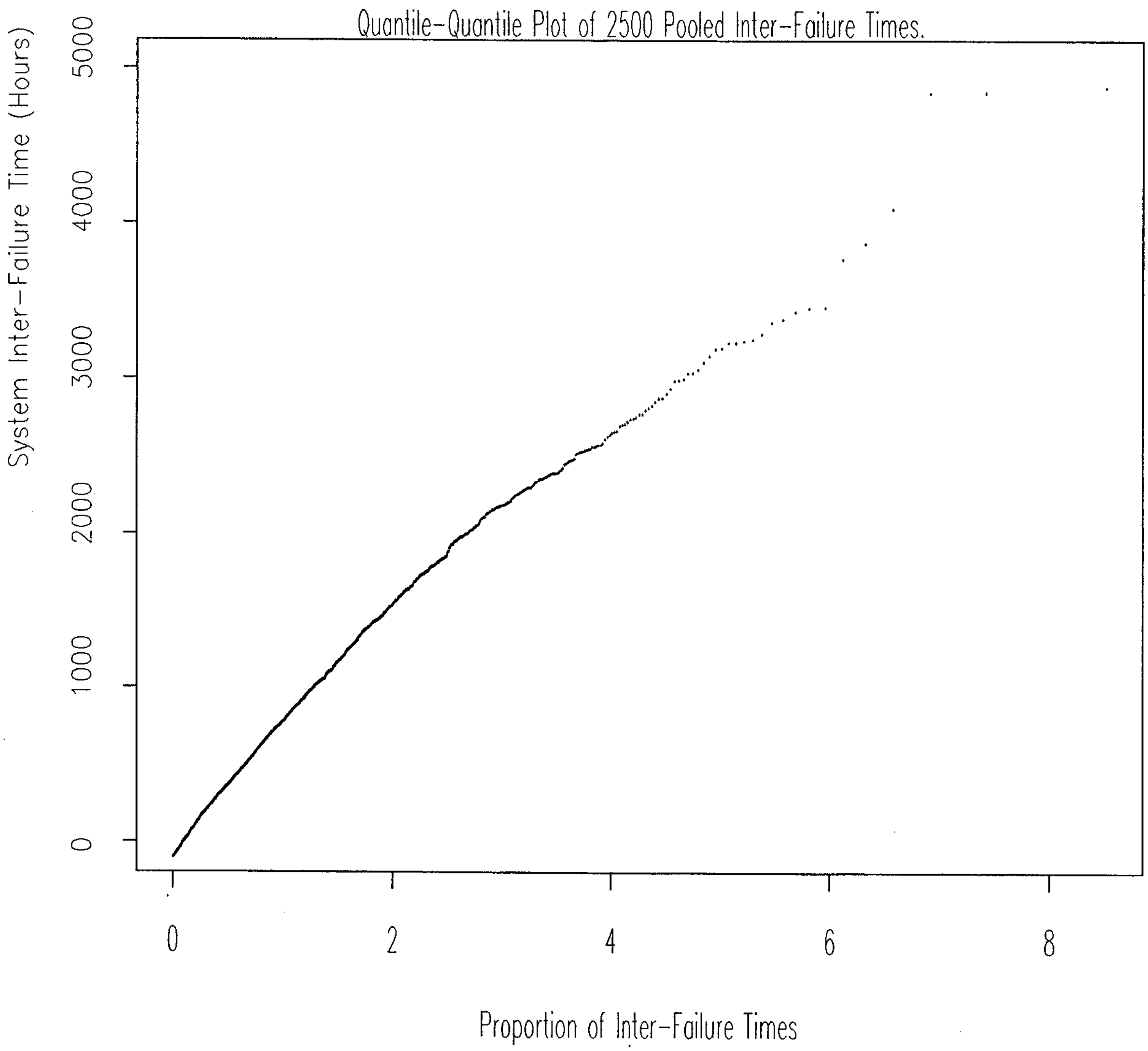

Figure 4.The Q-Q plot of the 2500 pooled inter-failure times plotted against the exponential distribution plots as a near straight line. This provides additional confirmation of the assumptions in Figure 3.

APPENDIX E. SUPERPOSITION OF TWO G32P SOCKETS

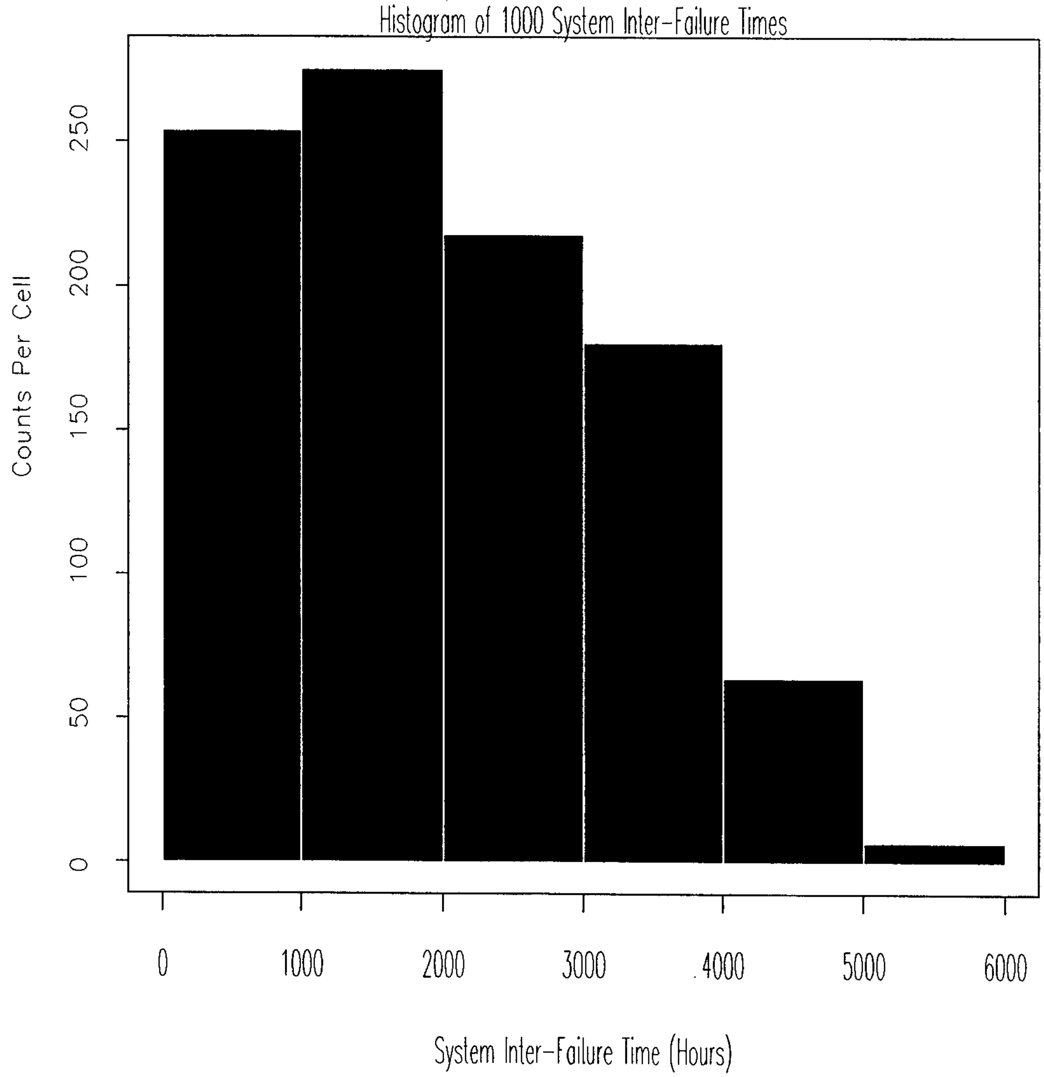

Figure 5. The histogram of two pooled G32P sockets does not indicate that the underlying distribution of the 1000 inter-failure times is exponential. Thus, we can not assume that the pooling of two sockets is a Poisson Process.

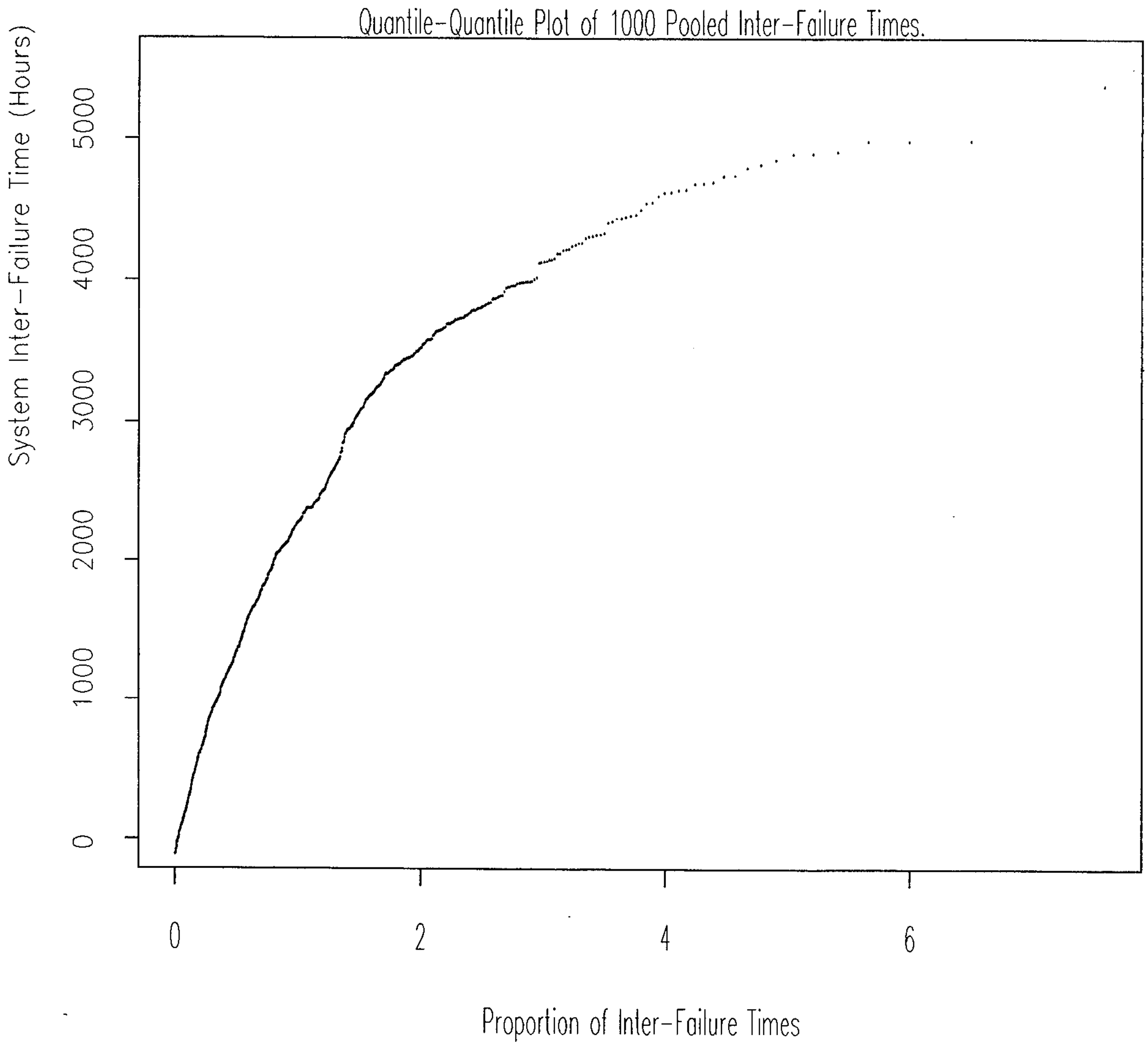

Figure 6. The Q-Q plot of two pooled sockets takes on a curved shape when plotted against an exponential distribution. This is clear evidence that the generated inter-failure times did not come from an exponential distribution.

APPENDIX F. LIGHT VALVE INVENTORY DATA

The following data is an inventory of light valve failure times, installed times, and spares as of October 6, 1995.

Site	Type	Failure Times		Survival Times		Spares
Beaufort	G43	831		1652		5
		1071		2992		
		4159		2697.		
		4116				
		4159				
Cecil Field	G32p	2896	2614	1383	5347	7
		3386	4629	1984	518	
		50	4300	3379	1842	
		8193		2278		
		4304		1573		
	G43	730		2265	2130	21
		3700		7210	1842	
					3442	
Cherry Pt.	G32p	955	4633	30	278	7
		937	4170	1501	925	
		1321	4250	1495	832	
		4848	0	2178	2100	
		4773	5034	3245	2517	
		4880	4019	5185		
		4470	0	2623		
		4527		896		
	G43	2097		50	3076	9
				3342	1	
El Toro	G43	4646		4537	1070	20
				2815	2420	
				2554	0	

Table 1. The table list light valve experience data for all sites which report quarterly data to ASO.

Site	Type	Failure Times		Survival Times		Spares
Lemoore		4100	4150	3924		0
		4080				
	G43	4100	4320	3369	2435	8
		4438	4150	1022	317	
		4080	4000	655	126	
		4200	4280	105	663	
		4150	4160	1280	71	
		4020	4220	1371		
Mirimar	G32P	4085	5372	2089		19
		4977	5977	979		
		5218	1271	582		
		3558	4947	2244		
		3522	2160	1274		
		4573	4500	1345		
		4111	5641	2227		
		5111	5046	2084		
		3899	4928	3380		
		4156	4000	1481		
	G38	3729		944		9
		4479		1589		
		4090		1480		
		4613		1538		
	G39	4176	4100	1538		6
		2526	4613	1489		
		3729		1074		
		2915		944		
Oceana	G32P	330	3940	1240	1719	7
		2350	2900	809		
		3940	440	2253		
		4035		716		
	G38	3787	3877	1360		5
		3663	3732	1502		
		3400				
	G39	3663	3877	1300		4
		3787	1682	2884		
Yuma	G32P	5000	2000	4048	255	21
		5200	0	2400	164	
		4863	3280	206		
		5000		153		
		5200		134		
	G43	1850		1028		6
		2500		1536		

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